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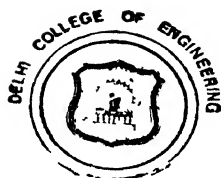
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PRINCIPLES OF  
ELECTRICAL ENGINEERING

## THE COMPANION VOLUME TO THIS BOOK

*By the Same Author*

### "TEST PAPERS AND SOLUTIONS ON ELECTRICAL ENGINEERING"

The above book, although issued separately for the convenience of the reader, is an integral part and an extension of "Principles of Electrical Engineering". It contains the full solutions to the 203 Test Questions included at the end of the present volume, and constitutes a valuable guide to the actual working out of numerical and other examples, without which a real grasp of the principles of electrical engineering cannot be obtained.

The examples are all chosen and arranged with a view to emphasizing the different aspects of the principles dealt with in the individual chapters of the present volume and to make clear any points which might otherwise be obscure. Wherever possible, the questions and answers have been taken from actual practice.

Publication of "Test Papers and Solutions" as a separate volume allows the reader to have before him the answer to the problem he is studying and, at the same time, to be able to refer to various sections of the text which may have a bearing on it.

# PRINCIPLES OF ELECTRICAL ENGINEERING

A Comprehensive Work covering the Principles of Heavy current and Light current Engineering Practice: also covering the requirements of the B.Sc. (Engineering), A.M.I.E.E., and Higher Examinations in this Subject.

*By*

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## PREFACE

THE aim of this book is to present as comprehensively and in as limited a space as may be possible, an account of the basic principles of the science of electrical engineering, a leading idea throughout the book being to place emphasis on the identity of the principles relating to both heavy current and light-current engineering practice. The scope of modern electrical engineering is immense and it is only when a clear understanding of the fundamental principles has been obtained that sound progress can be hoped for in the study of any branch of the subject.

Chapter I includes a brief historical survey of the development of standards for units of measurement from the earliest days of the industry, and it is hoped that students and others will find this useful as evidence of the vast amount of work which has been done in electrical engineering to make precision measurements possible. It is of some importance to note that in 1933 the "General Conference on Weights and Measures" resolved that from January 1, 1940, the technical units were to be based on the absolute C.G.S. electrical units, and this resolution would normally have come into force throughout the civilized world on that date: the immediate practical consequences of this change over are indicated on page 15. No account of units would be complete without including the M.K.S. system, the technical significance of which has become of rapidly increasing importance in recent years.

Chapters II and IV include an account of recent developments in insulation technique, and Chapter V gives a treatment of networks which it is hoped will be found useful for the solution of a wide range of technical problems.

In Chapter VII a necessarily brief account is given of some of the more important recently developed magnetic materials for both high-frequency and low frequency purposes, as well as for the manufacture of permanent magnets.

In Chapters IX and X the use of complex quantities is explained as well as their applications to a wide range of problems, and for this purpose it has been thought advisable to make use of German script letters to denote such complex quantities. The extreme simplification to which many otherwise complicated problems can be reduced by this method of treatment, should make it more widely used than appears to be the case at present.

Bridge methods of measurement in both heavy and light current work have now become of great technical significance, and several representative types of such bridges have been selected for description.

Oscillatory systems have necessarily received treatment in consider-

able detail in Chapters IX and X in view of the multitude of the varieties of electrical engineering machines and apparatus with which such problems are now associated.

In Chapter XII a number of graphical methods for the solutions of technical problems are explained, and these methods will often be found to provide a simple, accurate, and powerful means for solving problems for which mathematical treatment would be impracticable. The free and confident use of such methods, when appropriate, will frequently lead not only to a simple and exact solution, but will also provide a valuable insight into the physical significance of the intermediate stages by which the solution has been reached.

Chapter XIV contains a detailed treatment of "skin effect", and the attractive prospects of applying this phenomenon to a variety of industrial purposes as are now in prospect will give rapidly increasing importance to a study of the substance of this chapter.

The principles of the long distance high tension transmission of electric energy are considered in Chapter XV and should provide a sufficient foundation for those who wish to pursue in greater detail investigations in this branch of electrical engineering.

The basic facts and fundamental mathematical equations relating to the propagation of electromagnetic waves through space are given in Chapter XVI, and a knowledge of these is essential for any serious scientific discussion of radio transmission and reception.

It is realised that only by the actual working out of numerical and other examples can a real grasp of the subject be obtained, and for this purpose special attention has been given to the preparation of a set of examples associated with the contents of each chapter. The full solution for each example is given, for the reader's convenience, in a separate volume. In the case of a few of these examples the opportunity has been taken to include with the solution additional information to supplement the book work.

It is desired to make acknowledgment here of the courtesy of the Editor of *Engineering* in permitting the inclusion of material taken from a number of articles by the author and which have been published recently in that journal; references are given in the text of the book to the dates of issue of the articles concerned. Messrs. Methuen have also kindly agreed to the reproduction of some diagrams and other matter taken from the author's book, *Electrical Engineering*, which is now out of print.

T. F. W.

SHEFFIELD, 1946.

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# PRINCIPLES OF ELECTRICAL ENGINEERING

## Chapter I

### FUNDAMENTAL UNITS : TECHNICAL UNITS

THE fundamental units of length, mass, and time, which are now universally adopted for scientific purposes, are those which were agreed upon by a Congress in Paris in 1881, viz.

- UNIT OF LENGTH : One Centimetre (very closely equal to  $10^9$  times a quadrant of the earth measured from the Equator to the Pole).  
UNIT OF MASS : One Gram (very closely equal to the mass of one cubic centimetre of water at the temperature of its maximum density).  
UNIT OF TIME : One Second ( $\frac{1}{86400}$  part of a mean solar day).

It was agreed that units founded on these quantities should be called centimetre-Gram-Second absolute units or more briefly, C.G.S. units. These units are sometimes much smaller, and, in other cases, much larger than the quantities which they are required to define, and in order to avoid the necessity of having to use very large numerical multipliers or divisors, the Congress agreed that multiplication by one million could be expressed by the prefix "mega-", and division by one million by the prefix "micro-".

#### Mechanical Units

- FORCE : The c.g.s. unit is the *dyne*, and is the force which is required to give an acceleration of 1 cm. per sec. per sec. to a mass of one *gram*.  
WORK AND ENERGY : The c.g.s. unit is the *erg*, and is the work done when the point of application of a force of one dyne is moved through one centimetre.  
POWER : The c.g.s. unit is one erg per second.  
HEAT : The c.g.s. unit is the *gram-calorie* and is the heat necessary to raise the temperature of one gram of water through  $1^{\circ}\text{C}$ . (actually from  $4^{\circ}\text{C}$ . to  $5^{\circ}\text{C}$ ., that is, at the temperature of the maximum density).

Some useful approximate relationships between the magnitudes of units are given in the following list :











that is to say, the velocity  $c$  is that with which each wire must move in the direction of its length in order that the two wires shall have no action on each other.

### The Electrostatic C.G.S. Absolute System and the Electromagnetic C.G.S. Absolute System of Units

If, for open space, it be assumed that the dielectric constant is  $\epsilon = 1$  and the units of length, mass, and time are, respectively, the centimeter, gram, and second, then the electric and magnetic quantities when expressed in these units are said to be defined in *electrostatic c.g.s. absolute units*. Thus, the electrostatic c.g.s. absolute unity of quantity is that quantity which, when concentrated at a point distant 1 cm. in air, from an equal concentrated quantity, is repelled with a force of 1 dyne.

Similarly, if for open space the value of the magnetic permeability is  $\mu = 1$  and the units of length, mass, and time are the c.g.s. units, then the electrical and magnetic quantities expressed in terms of these units are said to be defined in *electromagnetic c.g.s. absolute units*.

For most technical purposes it is convenient to use multiples of the absolute units, that is, powers of ten, and in Table II are given the technical units and their relationships to the c.g.s. absolute units for some of the more important electric and magnetic technical quantities. It is of interest to note that the names *Ohm*, *Volt*, and *Farad*, were proposed by the British Association and officially accepted by the Paris Congress in 1881, this Congress also having settled the names *Ampere* and *Coulomb* for the technical units of current and electric quantity, respectively. The name *Henry* for the technical unit of inductance was determined by the Chicago Conference in 1893. Previously, the unit of inductance had been termed the *Quadrant*, since, in the electromagnetic c.g.s. absolute system, it has the value  $10^9$  cm., and this is also very approximately, the length of a quadrant of the earth measured from the Equator to the Pole. Another term which had been used for the unit of inductance was the *Secohm*, since the dimensions of the unit of inductance are

$$\text{Henry} = \frac{\text{Volt} \times \text{Second}}{\text{Ampere}} = \text{Second} \times \text{Ohm}.$$

It is of interest to note from Table I that electric resistance in the electromagnetic system of units has the dimensions of a velocity, whilst the dimensions of (electric current)<sup>2</sup> are those of a force. It will also be seen from Table II that the ohm is equal to  $10^9$  electromagnetic c.g.s. units, that is to say, the unit of velocity will be  $10^9$  of the c.g.s. unit. In other words, if the resistance of 1 ohm were to be taken as the absolute unit of resistance in place of the electromagnetic c.g.s. unit, the corresponding unit of velocity would be  $10^9$  times that of the c.g.s. unit.

Suppose, then, that the resistance of 1 ohm is taken as the new



### Absolute and Technical Systems of Measurement

In the foregoing considerations the abstract definitions of the absolute e.g.s. electric and magnetic units have been explained. The measurement of quantities as defined by such an absolute system may be achieved in two different ways, viz.

(i) The value of the quantity under consideration can be obtained solely by means of the units of mass, length, and time, and one electric or magnetic quantity, such, for example, as the calculated inductance of a coil of known dimensions.

(ii) A standard of reference may first of all be prepared, which shall be as nearly as is practicable of the same magnitude as defined by the corresponding absolute unit, and the quantity concerned can then be compared with this standard of reference. Such a system of measurement may be termed a *relative*, or *technical*, or *practical* system.

Until the last few years, the procedure (i) has presented such formidable difficulties as regards the measurement of a quantity in absolute units that it was not practicable to apply it, and, in consequence, the method (ii) has always been employed in practice. As will be pointed out in what follows, however, the precision with which the required measurement can now be made in absolute units is such that the Conférence générale des Poids et Mesures which, before the War, met once every six years at Sévres, resolved that on January 1, 1940, measurement in absolute units was to be the legal method in all those countries which had sent representatives to the "Metre Conference", and this comprised practically the whole civilised world.

For the *technical* system of measurement as hitherto carried out, it is necessary to have some concrete standards of reference which embody the respective absolute units as nearly as it is practicable to do so. Even before any absolute system of units was evolved, the rapidly developing electrical engineering industry felt the need for some authoritative standard of resistance, and those firms which were concerned with the manufacture of machines and apparatus had constructed arbitrary standards of reference, although none of these satisfied the required conditions of reliable stability. \*For example, about 1841 Jacobi proposed the first concrete standard for the unit of resistance, viz. a coil of wire of known length and cross-sectional area. This, however, was not satisfactory as it could not be duplicated with any degree of precision.

In 1860 Sir W. Siemens pointed out that, in order to overcome the difficulties due to non-homogeneity of solid metal, *mercury*, which is the only metal which is liquid at normal temperatures, should be used, and, owing to its homogeneous characteristic, stands alone as being of the same and unalterable specific resistance, so that in duplicating such a standard, it was only necessary to define precisely its geometrical dimensions (length and cross-section). This proposal was recognised as such an outstanding advance in comparison with all previous proposals

that it soon displaced them all. The proposal therefore made it possible to place the practical unit of resistance on a firm and reliable foundation. Siemens' proposal for the dimensions of such a mercury column was that the length should be 1 metre and the cross-section 1 square millimetre, and for a long period of years this was known as the "Siemens Unit".

In 1863 (Newcastle) the Appendix "C" of the Second Report of the Electrical Standards Committee by Clerk Maxwell and Fleeming Jenkin was issued. This Appendix dealt with the elementary relationship between the electrical quantities and is the historic achievement of Maxwell and the founders of the c.g.s. system. During the period 1862-75 a Board of Trade Committee made efforts to produce a more reliable standard of resistance from various alloys, of which the platinum-silver alloy was the most successful, and this was used for a long time and was known as the B.A. Unit. It was found, however, that this did not maintain its constancy over a long period of time and could not be duplicated with sufficient accuracy.

During 1888-90 E. Weston and Dr. Heusler collaborated in a systematic search for a suitable alloy, and eventually produced "manganin", which satisfied the most rigid requirements, and in consequence of which it is, even to-day, in an unassailable position throughout the world as a material for precision and standard resistances.

The position with regard to the evaluation of the resistance unit was not so satisfactory. Up to the year 1881 of the various methods for measuring the value of the material standard in absolute units, only four results were available, viz.

<i>Year</i>	<i>Observer</i>	<i>Magnitude of the Ohm expressed as the Length of a Column of Mercury of 1 sq. mm. Cross-section</i>
1873 . . . . .	Lorenz	1 0710 metres
1877 . . . . .	H. F. Weber	1 0590 "
1878 . . . . .	Rowland	1 0616 "
1881 . . . . .	Rayleigh and Schuster	1.0598 "

As regards the second electrical unit (i.e. current), up to the year 1881 no absolute measurements had been reported which fulfilled the required conditions as they were then understood. Whereas up to that date efforts had been concentrated on the definition of (absolute) independent units, a new departure was now made. The aim now was not only the creation of units based on reliable foundations, but also to provide for their introduction into practice and, with this in view, the first Paris Congress of 1881 established the Ohm, the Ampere, and the Volt, in place of the absolute units. The experience which had been

accumulated with regard to the choice of suitable resistance material was now crystallised by the resolution of the Congress that a mercury column of stated dimensions should be chosen as the unit of resistance, but a statement as to what those dimensions should be was postponed and a commission was charged with the task of preparing recommendations regarding these dimensions. (It is of interest to note here that the Congress of 1881 introduced the prefixes "mega-" and "micro-".)

The Paris Congress of 1884 fixed the value of the technical ohm, in accordance with the existing state of knowledge, as 1.06 Siemens units. A definition of the ampere was also given, and in order that the technical (empirical) unit could be easily and satisfactorily duplicated, it was defined by the amount of silver deposited in a silver voltameter by a quantity of electricity of 1 coulomb in 1 second. A statement as to what that amount of silver should be was deferred, since at that time sufficiently rigorous measurements of the electrochemical equivalent of silver had not been made. In addition to the two units of ohm and ampere, the joule and the watt were defined by the Congress, as well as the unit of inductance for which the term "Quadrant" had been introduced by Maxwell. This was replaced by the term "henry" at the Chicago Conference of 1893. Whilst at the time of the Paris Congress there was not sufficient experimental data available of the required precision, in the few years following, the number of exact determinations of the ohm and the ampere had accumulated to such an extent that adequate material was available for stating the absolute values.

The assessment in terms of the mercury column of the absolute ohm on the basis of the many investigations which had been made for this purpose, was a matter of some difficulty, and it seemed to be desirable to institute an expert critical survey of the results of the individual measurements in order to arrive at the most probable value, and the governing body of the P.T.R. (Physikalische technische Reichsanstalt) entrusted this work to Dr. Dorn, who was particularly well qualified to undertake it. The result of his examination of the fifteen investigations which came under review gave the probable value of the absolute ohm as equal to the resistance of a mercury column between 1.06274 and 1.06292 metres long, and one sq. mm. cross-section at 0° C. In view of the degree of uncertainty which these margins of values defined, Dorn recommended that the value should be fixed with an accuracy of 1 part in 1,000 and for this reason he proposed the value of 1.063 metres as the German Legal Ohm, it being observed that this value had already been proposed as the British Legal Ohm.

As regards the absolute value of the ampere, the matter was much simpler. For the purpose of fixing the legal value the P.T.R. selected the determinations of Lord Rayleigh and Mrs. Sidgwick for one basis of measurement, and that of F. and W. Kohlrausch for the other. In each case the determination was made by measuring the amount of

silver deposited in a silver voltameter by one ampere, the former value being 1.11794 milligrams and the latter 1.11826 mgm. As in the case of the ohm, it was not considered desirable to assume an accuracy greater than 1 part in 1,000, and the value was fixed as 1.118 mgm., it being observed that this value had also been decided upon by the British Board of Trade Committee. It is noteworthy that the mean value of all the reliable measurements which were available at that time was 1.118 mgm., and of these measurements, five were made by means of the "current balance" and one by each of the following instruments, viz. Tangent Galvanometer, Sine Galvanometer and the Dynamometer, and although the value of 1.118 mgm. was specified as having an accuracy of 1 part in 1,000, it was realised that, actually, the value was correct to within 1 part in 10,000.

Having settled in this way the values of the absolute ohm and the absolute ampere with an accuracy of 1 part in 1,000, and decided that this degree of accuracy could be attained by technical measurements, it appeared that the time had arrived at which international agreement could be established with regard to the legal definitions of the technical units.

### **The International Conference on Electrical Units in Charlottenburg, 1905**

At the suggestion of the Bureau of Standards, Washington, the P.T.R. issued invitations to those States which were concerned with the supervision and solution of the problems associated with the establishment of the electric units. Invitations were also issued to those specialists who were, by reason of their experience in this kind of work, particularly qualified to advise on this problem. The State Laboratories which were represented were P.T.R. (Germany): N.P.L. (Great Britain): Vienna: and Brussels, and the technical specialists were Professors F. Kohlrausch, E. Mascart and B. Carhart. The Chicago Conference of 1893 had fixed the respective values of the three quantities which are related by Ohm's Law, viz.:

(i) **THE OHM.** The unit of resistance shall be what is known as the "international ohm", which is substantially equal to one thousand million units of resistance of the c.g.s. system of electromagnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 gm. in mass, and of a constant cross-sectional area, and of the length of 106.3 cm.

(ii) **THE AMPERE.** - The unit of current shall be what is known as the "international ampere", which is one-tenth of the unit of current of the c.g.s. system of electromagnetic units, and is the practical equivalent of the unvarying current which, when passed through a solution of silver nitrate in water in accordance with standard specifications, deposits silver at the rate of 0.001118 gm. per second.

(iii) THE VOLT —The unit of electromotive force shall be what is known as the “international volt”, which is the e.m.f. that, steadily applied to a conductor whose resistance is one international ohm, will produce a current of one international ampere, and is practically equivalent to 1.1434 of the e.m.f. between the poles or electrodes of the voltaic cell known as Clark’s cell, at a temperature of 15° C., and prepared in the manner described in the standard specifications.

At the meeting of the Charlottenberg Conference the most urgent matter under discussion was the lack of uniformity of electrical units in the different countries, which was largely due to the fact that the Chicago Conference of 1893 had fixed the respective values of the *three* quantities which are related by Ohm’s Law, whereas not more than *two* of these quantities should have been fixed by definition, the third being then determined by Ohm’s Law. The matter was dealt with by resolving that the value of the ohm and the ampere should be specified by resolution. As a standard of reference for e.m.f. the Cadmium cell was chosen instead of the Clark’s cell as previously used.

The question was then considered as to what values were to be assigned to the technical ohm and the technical ampere, that is to say, what were to be the dimensions of the mercury column which would represent the resistance of 1 ohm and what the amount of the silver voltameter deposit which would be produced by a current of 1 ampere in each case, with a margin of error not greater than 1 part in 1,000.

### International Conference in London, 1908

The definitions issued by the Chicago Conference of 1893 did not make the International Units distinct from the absolute values expressed by the c.g.s. electromagnetic system. The London Conference made this distinction and adopted as the two basic units: (i) the ohm defined by reference to the mercury column, and (ii) the ampere defined by reference to the silver voltameter.

### Washington Conference, 1910

At this date only the N.P.L. and the P.T.R. possessed a mercury column resistance, and in order to compare these with wire resistances, the values of which had been measured by comparison, with the respective mercury columns were then compared with the following result:

$$(\text{Ohm})_{\text{N.P.L.}} - (\text{Ohm})_{\text{P.T.R.}} = 1 \times 10^{-5}.$$

The mean value of both units in combination with the results of the silver voltameter tests was made the basis of measurement for the e.m.f. of the Weston cell, and this value was then specified as

$$1.01830 \text{ volts at } 20^{\circ} \text{ C.}$$

**The International Conference of the Advisory Committee for Electric Units and the International Committee of Weights and Measures, Paris, 1928**

The first matter which this Conference had to decide was whether the empirical international units defined by the International Conference of 1908 in London should be maintained, or whether they should be replaced by the so-called absolute units. In favour of such a change-over was the fact that the degree of precision with which the absolute units could then be measured had reached such a stage that it was about of the same order as that with which the empirical units could be measured, that is to say, about the same degree of precision with which it was possible to measure the resistance of a column of mercury of given dimensions and the weight of silver deposit in a silver voltameter. The single objection to the change-over which gave rise to hesitation was the fact that a difference of 5 parts in 10,000 was known to exist between the empirical ohm and the absolute ohm, and this was sufficiently large to involve a widespread readjustment of calibration and other consequences arising from the relatively long period of time for which the international ohm had been the legal standard throughout the world. The Conference fully appreciated this argument, but held the view that the great advantages which would accompany the change-over were sufficient to outweigh the objections, and accordingly they resolved to recommend that the change-over should take place.

It was decided, however, that the change-over should not take effect before the relationship which existed between the international and the absolute units had been determined with the necessary and attainable precision. For this purpose it was agreed that those State Laboratories which were adequately equipped should carry out the necessary experimental work in accordance with a programme devised by the Advisory Committee. As a result of these decisions, the Bureau of Standards at Washington has obtained the following results, viz.

(i) CURRENT BALANCE METHOD (*Journal of Research*, 1934) :

One Bureau of Standards International Ampere = 0.999928 Absolute Ampere.

The authors are of the opinion that this result differs from the true value by less than 20 parts in one million.

(ii) CALCULATED INDUCTANCE OF A SINGLE-LAYER SOLENOID (*Journal of Research*, 1938) :

One Bureau of Standards International Ohm = 1.000468 absolute ohms.

The authors are of the opinion that this result differs from the true value by less than 20 parts in one million.



















It will also be clear, however, that the constant  $4\pi$  will appear in some of the rationalised units, whilst it is absent from the corresponding unrationalised units. Thus, in both systems the relationship for open space, viz.

$$(\text{Permeability}) \times (\text{Dielectric constant}) = \frac{1}{c^2}$$

must hold, where  $c$  is the velocity of light, and for c.g.s. units,  $c = 2.9979 \times 10^{10}$  cm. per sec. and  $\mu = 1$ .

In the unrationalised electromagnetic c.g.s. system

$$\epsilon = \frac{1}{c^2} = \frac{1}{(3 \times 10^{10})^2},$$

whilst in the rationalised electromagnetic c.g.s. system  $\mu^* = 4\pi$ , so that

$$\epsilon^* = \frac{1}{\mu^* c^2} = \frac{1}{4\pi(3 \times 10^{10})^2}.$$

On the whole, therefore, it would appear that the rationalised system of units does not have any outstanding practical advantages as compared with the unrationalised system.





atom therefore has the simplest structure of all elements, viz. 1 proton of charge  $+1.59 \times 10^{-19}$  coulomb and 1 electron of charge  $-1.59 \times 10^{-19}$  coulomb revolving in an orbit round the nucleus, as is shown diagrammatically in Fig. 1. The atomic number of lithium is 3, that is to say, there are normally 3 electrons moving in orbits round the nucleus, and consequently the nucleus contains 3 protons. The atomic weight of lithium, however, is 7, so that in addition to the 3 protons the nucleus of the lithium atom contains 4 neutrons, the diagrammatical representation being shown in Fig. 2.

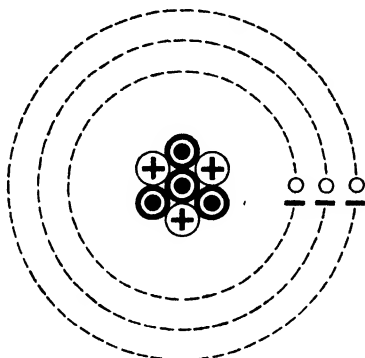


Fig. 2.

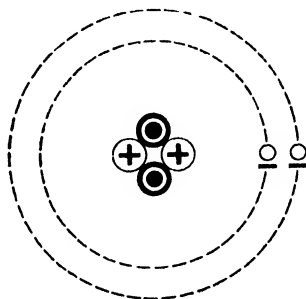


Fig. 3.

Another atom of simple structure which is of great technical significance (see page 29) is the helium atom  ${}^2\text{He}$ , of which the atomic number is 2, so that there are 2 electrons normally moving in orbits round the nucleus. The atomic weight of helium, however, is 4 times that of hydrogen, so that in addition to the 2 protons the nucleus of this atom contains 2 neutrons, as shown in Fig. 3.

### The Photon

The foregoing account of the Bohr-Rutherford structure of the atom as a series of electrons revolving round the nucleus in a similar fashion to that of the planets travelling in orbits round the sun does not suffice to account for the energy transformations which take place within the atom, and it is necessary to combine the Bohr view of the atom with the quantum theory, in accordance with which, of all the paths which are mathematically possible, only a few are actually available for the electron orbit.

In the normal condition of the atom, each of the revolving electrons will occupy the orbit of lowest energy of all its possible paths, and this is its basic orbit. It is only by exciting the atom by conveying energy to it from outside that the electron can be forced into an orbit of higher





been found that, during the transformations of the protons and neutrons, secondary electric particles, viz. the neutrino and the positron appear, but which apparently have only a transient existence.

The magnetic characteristics of the ferromagnetic metals iron, cobalt and nickel can now be accounted for as a consequence of the "spins" which are superposed on the electrons revolving round the nucleus, that is a motion similar to the revolution of the planets on their axes. Further reference to this will be found in Chapter VII.

### Conduction of Electricity through Gases

The flow of current in a metal conductor is defined by Ohm's Law, which relates the potential difference with the current, as is explained on page 44. This law, however, entirely fails when currents which are formed of free electrons or free ions are considered. For such currents in open space the controlling factor is the "space charge", and in place of the inexhaustible store of electrons which are available in a metal conductor, which in the case of currents formed by electrons in open

space it is first of all necessary to provide the electrified particles of which the electronic and ionic currents are composed.

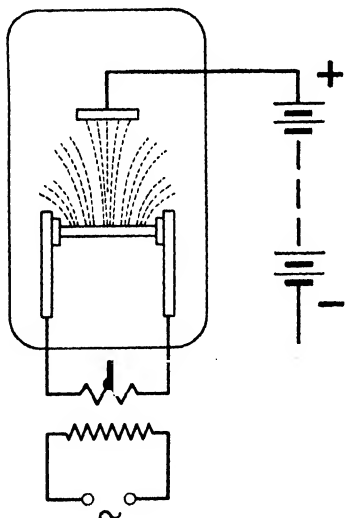


Fig. 6.

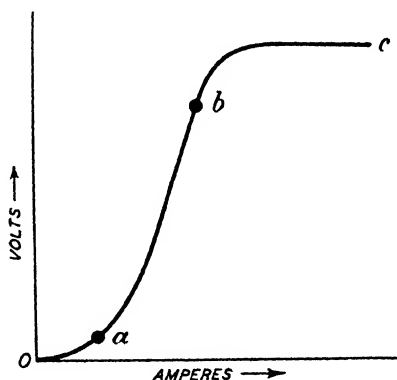


Fig. 7.

Suppose two electrodes as shown in Fig. 6 are respectively maintained at positive and negative potentials, and that the space surrounding them is completely exhausted of gas. If the temperature of the negative electrode, that is, the cathode, is raised to a sufficient degree, electrons will be ejected and will travel towards the anode under the influence of the electrostatic field which exists between the electrodes. At first sight





considered already in the foregoing. These electrons are accelerated in the electric field and, by ionisation of the gas, produce positive ions. The ions then travel towards the cathode, neutralising the negative space charge on the way, and so release those electrons in the neighbourhood of the cathode which are bound by the space charge, a state of equilibrium being eventually reached which is dependent upon the magnitude of the p.d. between the electrodes.

There is another type of glow discharge which is of great practical significance and which does not depend upon a heated cathode for the production of the electrons. This employs a cold cathode which actually produces a glow discharge very similar to that which is obtained from a hot cathode. This may be explained by means of the following considerations. In a highly evacuated chamber there will always be individual free electrons present which may be due, for example, to radio-active effects from which no such space will ever be entirely free. Such free electrons will become accelerated in the electric field between the electrodes and will ionise the gas. The released positive ions will then move towards the cathode with ever-increasing speed in the electric field and will also ionise the gas in their path. In this way a cumulative or "avalanche" effect develops at a very high speed until a condition of equilibrium is reached. The difference between the glow discharge produced by a hot cathode and that produced by a cold cathode is not so great as might appear at first sight since, fundamentally, it is a matter of indifference whether the positive ions release the electrons from their bound condition in the space charge, or from their bound condition in the atomic structure of the material of the cathode.

### Mechanism of the Glow Discharge

The addition of a minute amount of gas or vapour in the evacuated container vessel greatly complicates the process by which a glow discharge is produced. The electrified carriers newly released by ionisation partake in the further process of ionisation, and this process is the most important factor in the excitation of a glow discharge. In order that the electrons shall acquire sufficient energy to effect ionisation by impact with a molecule, they must travel across a definite potential difference in the electric field: that is to say, for a given field a definite distance must be traversed. In order to simplify the consideration as much as possible a highly idealised condition will be assumed in which the two determining factors are (i) The distance  $x$  which the electrons must travel when ejected from the cathode in order that they may acquire the requisite amount of ionising energy, and (ii) the so-called "mean free path"  $y$ , that is, the distance which the electrons must travel on the average before they encounter a molecule, this distance being dependent upon the gas pressure in the evacuated space.

Assuming that at the commencement of the process the potential



density, and consequently, for a given current, the space charge is locked with a definite surface area of the cathode. The consequence of this relationship is that the amount of the cathode surface which is covered by the glow discharge will increase proportionally with the current strength so long as the total surface of the cathode remains uncovered. That is to say, so long as the total surface of the cathode remains uncovered, the cathode pressure drop will remain constant at its "normal" value as the current strength is varied. If, after the whole cathode surface has become covered by the glow discharge, the current is still further increased, an increase of the cathode pressure drop will take place, and since there will be, in consequence, a greater number of electrons ejected per unit area of cathode surface than will be the case with the normal glow discharge, the whole character of the discharge will be abruptly changed. The negative glow will contract to a single brilliant spot which will move restlessly over the surface of the cathode, thus the glow discharge will have become transformed into an *arc discharge*.

A general idea of the physical sequence of events may be obtained as follows. Suppose that the glow discharge has completely covered the cathode surface, or that the current strength has reached its maximum value consistent with the normal cathode pressure drop. If, now, at some spot  $x$  on the cathode surface the current density is increased, this will cause an increase in the number of ions at the corresponding point in the positive ion space-charge layer which is opposite to the spot  $x$  on the cathode surface. The temperature at this point on the cathode surface will therefore rise and the electron emission will automatically concentrate at this spot, so that the glow discharge will be transformed into the arc discharge.

### Conduction of Electricity across a Spark Gap

In the case of air and other gases, the conduction of the current across the gap is effected by ionisation of the path and this may be briefly considered in the light of what has been said in the foregoing. Since the ionisation of the path is caused by the electric field, a certain time interval is required to produce the requisite conductivity, so that between the initiation of the ionisation and the subsequent spark discharge, a definite, though extremely small, time interval is required, and this interval forms the "discharge lag" of the spark. The duration of this time lag depends upon a complex system of relationships and may vary between wide limits such as  $10^{-4}$  to  $10^{-8}$  sec. In the case of homogeneous electric fields, the dissociation of the molecules and atoms of the gas into electrified particles will take place more or less simultaneously throughout the whole path of the subsequent spark, so the lag of the discharge in this case will be relatively small. In the case of non-uniform fields, however, the ionisation process will be initiated

at points of maximum field intensity and will spread from there outwards. When electrodes are used of which the radius of curvature is small in comparison with the length of the gap, the discharge lag will be greatly increased. For oil and other liquid insulators, the energy of ionisation is very large, and in such cases the discharge lag will be very much greater than in air for otherwise similar conditions. The same considerations also apply to solid insulators such as porcelain, paper, and mica. In the case of unsymmetrical electrodes such as are found in suspension insulators, the polarity has a great influence on the discharge. In general, the negative electrode may be said to exert the controlling influence since the electrons on which ionisation depends are supplied from this electrode.

### The Photoelectric Effect

If light falls on a naked metal surface, particularly one of the alkali metals, a simultaneous emission of electrons takes place from the surface of the metal, and the intensity of this emission is directly proportional to the intensity of the incident light of a given wave-length. This proportionality of the intensity of the incident light and the intensity of the stream of emitted electrons finds a valuable application for photometric measurements and a photoelectric cell constructed for this purpose is illustrated in Fig. 10. The sensitive metal surface (e.g. potassium) on which the light falls is shown at *C*, and a battery *B* is connected in series with an ammeter so that the negative pole is joined to *C* and the positive pole to the anode *A*. The p.d. applied between the anode *A* and the cathode *C* causes the emitted electrons from the cathode surface to flow into the anode whilst an equal number pass from the battery into the cathode. The range of operation of this cell extends from ultra-violet wavelengths and through the complete spectrum (see Fig. 4) down to the short infra-red wavelengths and particularly important application of this type of appliance is found in the measurement of light for astronomical work. Such light-sensitive cells are also applied to an immense variety of important industrial purposes.

### Cathode Rays

It has been seen on page 33 that an electric discharge through an exhausted tube produces a "cathode glow" at the negative electrode. In 1880 Crookes showed that when the gas pressure is reduced to a sufficiently small value, the glow in the tube disappears altogether and the passage of electricity is only shown by the fluorescence of the glass tube which becomes extremely brilliant at that part of the tube which faces the cathode. Crookes explained this by assuming that electrical particles were shot off the cathode at right angles to its surface and formed a beam or "ray" which he called in consequence the "cathode ray". It was found that a particularly brilliant phosphorescent glow was













































































Böning has shown that by defining the quantities  $a$ ,  $b$  and  $c$ , as respective functions of the temperature, the expressions (32) and (33) can be made to give results which are in accordance with the actual experimental data. In Fig. 32 are shown the calculated and experimental results for the rupturing pressure of "Frequenta".

Whilst there can be no doubt as to the validity of Böning's theory as applied to many insulation materials, it can hardly be applied without modification to substances such as glass.























































































































































































































































































































































































































































































































































































































































































































































































































































































































































































































